Backward Stochastic Differential Equations

Speaker : Nathaniel Cogneaux

May 19th, 9h30 in 1BC45

Abstract :

Throughout this Seminar I will present the basic theory of Backward Stochastic Differential Equations (BSDEs). They are stochastic differential equations (SDEs) with a terminal condition where the solution is required to be adapted with respect to an underlying filtration. BSDEs are commonly found in a variety of applications, including stochastic control, mathematical finance, and nonlinear Feynman-Kac formulae, as they arise naturally in these contexts.
Our problem starts as soon as we consider an equation as simple as the following :

$$\begin{cases} dY_t = 0 \\ Y_T = \xi \end{cases} \tag{1}$$

Here, ξ has to be $\mathcal{F}_T$ measurable, and since $dY_t = 0$ we also have $Y_t = \xi$ $\forall t$. So, in general, there is no solution to (1) that is $\mathcal{F}_t$-adapted. In fact, we will need to introduce a process to force Y to be adapted.

This seminar will provide you with a substantive introduction to BSDEs, including their well-posedness, basic properties, and significance, allowing you to develop a fundamental understanding of this important mathematical tool.

References :

• Zhang, Jianfeng (2017). Backward stochastic differential equations. Probability theory and stochastic modeling. Springer New York, NY.
• N. El Karoui, L. Mazliak (1997). Backward stochastic differential equations. Chapman and Hall/CRC Research Notes in Mathematics Series. Paris.